Are specific skills an obstacle to labor market adjustment?☆

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A B S T R A C T

This paper shows that specialized education reduces workers’ mobility and hence their ability to cope with economic changes. We illustrate this point using labor force data from two countries having experienced important macroeconomic turbulence; a large economy with rigid labor markets, Poland, and a small open economy with increased flexibility, Estonia. We find that holding a vocational degree is associated with much longer unemployment duration spells and higher likelihood of leaving activity for older workers. We then build a theoretical framework in which young agents’ careers are heavily determined by the type of initial education, and analyze the transition to a new steady-state after a sectoral demand shift. Quantitative exercises suggest that the over-specialization of the labor force in Poland led to much higher and persistent unemployment compared to Estonia during the period of EU enlargement. Traditional labor market institutions (wage rigidity and employment protection) lead to an increase of the unemployment gap, but to a lesser extent.

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1. Introduction

Macroeconomic shocks leading to sectoral reallocation might have long-lasting consequences, in particular for the labor market. Their effect varies considerably across countries: some economies seem to have a relatively good absorption capacity, while others face long periods of unemployment for reasons which are not always easy to identify. The traditional explanation for such marked cross-country differences in the response to shocks is the variety of institutions that govern modern labor markets (Blanchard and Wolfers, 2000). This paper shows both theoretically and empirically that, in addition to institutions, obstacles to labor mobility due to skill specificities are key determinants of the speed of labor market adjustment.

The underlying logic of our analysis is simple. Suppose that initial education determines the career choice of workers and notably the sector (or occupation) where they work. In this context, a sectoral reallocation shock leading to several industries or occupations becoming obsolete, will also imply the obsolescence of the workers with more specific skills. In the absence of sectoral mobility—say, when 55 year old coal miners are reluctant or unable to apply for waiter jobs in fancy restaurants—the speed at which the labor market adjusts is the rate of demographic turnover, arguably a slow adjustment mechanism.

To give a brief overview of the argument of the paper, consider the example of two economies—Estonia and Poland—having faced similar macroeconomic turbulence, both the announcement of enlargement to the European Union in 1998 and the sequel of the Russian crisis, and having diverged afterwards. As Fig. 1 shows, the labor market in each economy has evolved quite differently since 1998, with the unemployment gap widening dramatically from 0.7 percentage points in 1998 to almost 10 percentage points in 2002. Past education choices leading to the accumulation of sector and job specific skills explain a large part of such differences, and notably the high persistence of unemployment in Poland. Indeed, the proportion of employed workers who attended vocational schools is much larger in Poland than in Estonia: 2/3 vs. 1/3 approximately. This is only one part of the story however, since labor market institutions may favor or prevent sectoral mobility. Retraining policies as those available in Estonia might increase the rate at which workers allocate to the new emerging sectors. On the other hand, Polish stringent employment
The macro-labor literature has increasingly recognized that obstacles to the allocation of workers to jobs are crucial factors affecting the dynamics and the current level of unemployment. Blanchard and Katz (1992) have studied the adjustment of US states, notably Massachusetts having faced a large negative employment shock in the 80s, in a context of relatively flexible labor markets. Marimon and Zilibotti (1998) have shown that the dynamics of unemployment in eleven European countries were well accounted for by industry effects, where the Spanish case stands out as the transition from agriculture has been particularly costly. As regards to structural change, long-run trends of sectoral reallocation and their interaction with labor market performance in the presence of frictions are studied by Messina (2006) and Rogerson (2008).

The specificity of human capital has been central in very few papers. Rogerson (2005) presents a model of sector-specific skills in the Lucas–Prescott island tradition, and investigates individual trajectories of finitely-lived agents, with permanent non-employment after displacement being a possible outcome. Interactions between institutions and specific skills are discussed in Wasmer (2006), who studies the role of employment protection and frictions in promoting the accumulation of specific human capital investments, and argues that in steady-state, economies with specific skills do well, but during the transition in turbulent times, their labor force face high transitional costs. Similarly, Ljungqvist and Sargent (1998) incorporate human capital losses while unemployed in a model of search, and account for country-differences in unemployment due to greater or lower generosity of unemployment compensation. Albrecht and Vroman (2002) highlight the role of specific skills in the labor market. They present a model with low and high skill workers where the former can only do low-skill jobs. An application of this model is Albrecht et al. (2009), which evaluates a skill enhancing program in Sweden. In the transition literature, Garibaldi and Brixiova (1998) present a two-sector continuous-time model similar to ours and investigate the role of labor market institutions such as unemployment benefits. Flanagan (1998) and Boeri (2000) have stressed the role played by the specificity of their human capital in the rise of unemployment in transitional economies. In the trade literature, the consequences of structural change driven by trade liberalization and increasing magnitude of capital flows have often been addressed in models that assume away any labor market imperfections, thus having relatively little to tell about unemployment dynamics. Empirically, displacement has been used to measure the amount of specific skills embodied in workers’ careers by Kriechel and Pfann (2005), who estimate the impact of general and specific skills on unemployment duration and wage losses after displacement.

This paper claims that specialized education decreases workers’ mobility and hence their ability to cope with economic changes. Empirically, the paper shows that workers holding vocational diplomas from formal education have had a hard time in readapting their skills in a period of rapid economic transition in Poland and Estonia. The paper’s theoretical contribution to the literature is to provide a dynamic model of a two-sector economy with time-varying labor supply facing asymmetric shocks, and to study the various mechanisms of adjustment at work along the dynamics of transition to the new steady-state. The model is exploited to match the unemployment experience in Poland and Estonia during the enlargement process, an exercise that helps disentangling the relative importance of skills’ specificity and institutions in driving the divergent outcomes in these two countries.

After a brief description of the macroeconomic context in Poland and Estonia in Section 2, Section 3 measures the costs of reallocation in the presence of specific skills. For this purpose we analyze the dynamics of employment and unemployment using micro data for these two economies from the labor force surveys. Our empirical results indicate that age, tenure and above all, vocational initial education are associated with higher unemployment duration and a higher likelihood to exit the labor market among the older workers. Interestingly, these patterns are present in both countries, suggesting that cross-country differences in the responses of labor markets to similar macroeconomic shocks might be accounted for by the differences in the total stock of specific skills highlighted above, or differences in institutions. The main lesson of this empirical analysis is that the specificity of skills is an obstacle to reallocation and can be a serious macroeconomic issue. In this respect, the mechanisms studied here are applicable to several other macroeconomic experiences.

In Section 4, we model reallocation of specialized labor across sectors following a relative demand shock in a two-sector Mortensen–Pissarides framework with wage rigidity and endogenous job destruction, augmented with specific human capital in which young agents initially are allocated into vocational or general education. The contribution to the literature here is to solve for out of the steady-state equilibria in continuous time and to characterize the saddle-path dynamics of its four predetermined variables and its four jump variables. We therefore provide a methodology to obtain the numerical resolution of the associated system of ordinary non-linear differential equations, which may be adapted to any continuous-time matching model as an alternative to discrete-time dynamic macroeconomic models. This allows us to analyze the transition to a new steady-state when one of the sectors expands and the other declines. We find three different time horizons in the transition: i) an initial and instantaneous period of increase in unemployment, as firms in the declining sector immediately layoff a sizeable fraction of the labor force; ii) a relatively rapid period of recovery—about 2 to 5 years—in which firms, facing a large pool of unemployed workers, post more vacancies; iii) a very slow period of convergence, due to mismatch between demand and supply of skills across sectors. In the absence of labor mobility, our model indicates that the period of convergence to a

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1 This may be what happened on the chart over the period 1993–1998: the decline in Polish unemployment seems to be in part associated with early-retirement policies while the increase in Estonian unemployment is related to drastic trade liberalization occurring at that time.

2 The main difference is that in their paper there is a transition from the public sector to a private sector, whereas we consider two private sectors. Compared to them, we also model endogenous job destruction and account for the skill composition of the labor force.

3 For a few exceptions see Learner (1980), Feenstra and Lewis (1994) and Saint-Paul (2005).

4 In this version of the paper we did not estimate wage equations, but focused on unemployment dynamics and labor market transitions. Kriechel and Pfann (2005) use information on higher ranks in the hierarchical position of displaced workers (such as managerial experience) as a proxy for general skills. We use the nature of diplomas as proxies for the degree of generality of skills as our focus is on the interaction between education and labor market adjustments.
steady-state with no mismatch is of the order of magnitude of a generation, i.e. the necessary time for older workers with inadequate skills to have retired. 

Section 5 outlines quantitative exercises to disentangle the respective role of the stock of specific skills, labor market institutions, training and early retirement policies in the divergent unemployment paths observed in Poland and Estonia during the years following the announcement of enlargement and the Russian crisis. Section 6 concludes.

2. Economic and institutional context in Estonia and Poland

The past decade in eastern European countries constitutes a particularly interesting period to document the importance of specific skills with regards to labor market adjustments. Countries faced important reallocation shocks, leading to the decline of obsolete industries and firms and the expansion of a new modern private sector. During this time, the international trade pattern of eastern European countries had to adjust rapidly: trade with the former Soviet Union declined substantially, especially after the Russian crisis, and integration to the west became increasingly important. All this took place at a relatively high speed and presented an unusual large scale. Hence, this period constitutes an interesting laboratory to capture the effects of a large reallocation from a declining private sector to a modern private sector on individuals’ labor market performance, and to investigate its macroeconomic implications in the presence of imperfect labor markets. We will study this period in restricting our analysis to the late 1990s and early 2000s, since we do not want to capture the transition to the market economy that followed the collapse of the centrally planned economies, a phenomenon extensively studied in the literature. Poland and Estonia further possess two interesting features for our study. First, and in line with most previously centrally planned economies, the education system in both countries has been traditionally oriented towards the provision of skills. However, the share of workers with vocational education in the working population is much larger in Poland than in Estonia. Second, both differ to a large extent in their labor market institutions. Estonia is often presented as an example of flexible labor markets, even among an enlarged EU, while the Polish labor market is characterized as rigid, offering generous safety nets and early retirement policies, and featuring stringent employment protection laws. See Lamo et al. (2006) for further discussion of the institutional characteristics of these two countries.

3. Empirical evidence

This section assesses the different fortunes of workers with different skills during this reallocation process. For this analysis we rely on the labor force surveys in Estonia and Poland over the period 1997–2003. We shall proceed in two steps. We first estimate various unemployment duration models, where we investigate the impact of the type of education on the ability to re-enter employment from unemployment. Then we estimate transitions for older workers with different skills from employment and unemployment into inactivity, aiming at capturing another adjustment mechanism that we expect to differ across educational diplomas: the incidence of early retirement.9

3.1. Data description

The Estonian and Polish labor force surveys (ELFS and PLFS respectively) are relatively homogenous, very similar to the labor force surveys carried out in the other EU countries, and by most standards the best household surveys among the former eastern European countries. Both surveys contain standard demographic and job characteristics, are run quarterly, and their longitudinal nature allows to follow individuals for a maximum of 1.5 years, tracing their relevant labor market spells on a monthly basis.9

Table 1 shows some summary statistics for our sample of employed workers excluding self-employed and part-timers, as well as for the sample of unemployed and inactive individuals. First, note that the dispersion of wages is higher in Estonia than in Poland. Second, population is slightly older in the Estonian data, but tenure is on average three years lower (6.9 vs. 9.9 in Poland). Third, the level of education, measured by the average years of schooling is large in both countries (12.6 and 13 among the employed in Poland and Estonia, respectively), higher than in many EU-15 economies such as France, the Netherlands and Spain (OECD, 2006). Fourth, in both countries the share of workers with vocational education (here divided between basic and secondary vocational) is high when compared with the EU-15 countries, but is almost double in Poland than in Estonia (63% against 35% of the employed population when considering basic and secondary vocational together). Fifth, there is an over-representation among the employed of individuals of Estonian origin (as opposed to Russian) in Estonia. Sixth, in both countries workers with secondary vocational education are over-represented among the employed, as compared to the unemployed. The opposite is true for workers with basic vocational education.

3.2. General strategy and workers’ heterogeneity

There is a great variety of vocational educational degrees in Estonia and Poland, and the contents of curricula and length of studies has changed over the years. Hereafter, our strategy is to control for the degree of specific skills with a dummy variable reflecting whether individuals’ highest degree of education is basic vocational or secondary vocational. Since the counterpart, basic general and secondary general degrees, do not necessarily have the same exact number of years of education—they may differ by one or two years in one direction or the other—all our specifications will additionally control for the number of years of schooling.10 We have explored different specifications including and excluding the agricultural sector (much larger in Poland), and results are virtually unchanged. Hence, we present our specifications for total private employment.

A relevant issue for the empirical analysis is whether there is self-selection into the type of education. If this was the case, the observed

5 Other experiences such as trade agreements (NAFTA, various rounds of the World Trade Organisation), have only progressively removed trade barriers and are limited in scope. In contrast, the 2004 enlargement implied, at the time it was officially agreed on in 1998, the accession of several new countries with the complete removal of trade barriers in a short time horizon, covering all sectors of activity. Further, most western European countries have kept strong barriers to control migration flows from the East. Thus, the impact of the enlargement in these countries is mostly a reallocation of labor within the new EU Member States.


7 Also in line with what happen in other previously centralised European countries, enrollment rates in vocational education have decreased in Estonia and Poland since the collapse of communism. This is an indication that there was indeed over-specialisation under central planning. In 2004 the share of secondary students enrolled in vocational over total secondary enrollment decreased to about 35% from almost 45% in 1990 in the case of Estonia. These numbers are 78% in 1990 and 58% in 2004 for Poland.

8 In a previous version of this paper, we also provided evidence that shows that wage losses of displaced workers were higher among those with specific skills. For the sake of concision this part has been removed in this version, but the reader can refer to Lamo et al. (2006). Also see Kriechel and Pfann (2005) for a study on the respective role of general and specific skills on wage losses.

9 Other data on the ELFS are run quarterly after 2000q2. However, the 1999 and 1998 annual waves allow to follow individuals for a maximum of 1.5 years, tracing their relevant labor market spells on a monthly basis.

10 The reference group for basic vocational consists then of workers with the same number of years of education who attended a non-vocational school. A similar interpretation will be given to the coefficient of secondary vocational.
outcomes such as wages, separation rates and hazard rates could reflect personal characteristics that we are not able to account for (e.g. talent, IQ, social origin, etc.). In particular, if less able individuals were sent to vocational education during the communist times, worse labor market outcomes today might reflect unobserved characteristics rather than the lack of adaptability of their skills. However, the available evidence indicate that workers with vocational education were not necessarily the least talented individuals: although evidence on returns to schooling during the pre-transition period is scarce, Rutkowski (1996, Table 5) reports returns to different educational diplomas in Poland in 1985, and shows that workers with basic vocational education earned 8 percent more in 1985 than workers with secondary or post-secondary education, which instead had between 1 and 3 years of schooling more than the former group. Moreover, the returns to education were relatively flat, the wage premium of workers with a university degree being only 16% with respect to workers with vocational education. Chase (1998, Table 5b) finds that in the Czech Republic and Slovakia the returns to secondary technical education were higher than the returns to secondary academic education both for men and women, by 1984. Similarly, Flanagan (1998) documents the overvaluation of vocational education in the Czech Republic during communism. If anything, this would suggest that more, and not less, able workers would self-select into vocational education prior the transition. To our knowledge, no comparable study exists for Estonia.

There is evidence suggesting that during the first years after the transition to market economies returns to vocational diplomas in the Czech Republic, Hungary, Bulgaria and Poland were lower than returns to general education (Boeri, 2000). This however is not necessarily a sign of low unobserved ability of workers with vocational qualifications, but on the contrary consistent with our claim that there is a penalty associated with specific skills at times of macroeconomic turbulences/large reallocation shocks. Nonetheless, we will attempt to control for unobserved heterogeneity in the analysis of unemployment duration presented below.

### 3.3. Unemployment duration

This section assesses the costs of unemployment in the presence of labor market imperfections, and in particular, in the presence of specific skills as opposite to general education. We investigate the determinants of unemployment duration after job separation. We construct a monthly data set with all relevant labor market spells for each individual observed in the sample during 1997–2003. For the analysis, we include all workers experiencing at least one unemployment spell after job separation. The samples are right censored, and this will be accounted for in the estimations.

The first feature that clearly stands out from the data is that unemployed workers find a job faster in Estonia than in Poland. For example, two years after job separation, over 27% of workers who experienced an unemployment spell are still unemployed in Poland, while this number is only 14% in the case of Estonia. To understand the determinants of the so called survival rates into unemployment and notably to understand the role of vocational education in re-entering employment, we estimate a multivariate model with observable and unobservable heterogeneity. The hazard rate is the probability of re-entering employment after job separation, conditional on having experienced an unemployment spell. Let the duration of the unemployment spell after separation be described by the density and distribution functions; f(t) and F(t). Then, the survivor function is defined as: \( S(t) = 1 - F(t) \), and the hazard function as: \( h(t) = \frac{f(t)}{S(t)} \). We estimate a proportional hazard model: \( h(t) = h_0(t) \exp(\beta'x) \), where \( h_0(t) \) is the parametrically specified baseline hazard, \( x \) is the vector of explanatory variables and \( \beta \) the vector of coefficients. We use a Gompertz-distributed baseline hazard for Estonia and a Weibull in the case of Poland, and allow for unobservable heterogeneity by using a mixture distribution where heterogeneity is represented by a gamma function.\(^{11}\)

Compared to the sample statistics in Table 1, we exclude new entrants and re-entrants into the labor force. As a result, the duration analysis includes a maximum of 4867 unemployment spells after job separation for Estonia and of 23,006 spells in the case of Poland. All results of the duration analysis are presented in Tables 2 and 3 for Poland and Estonia, respectively. Column 1 provides coefficient estimates for the basic specification, where the covariates include measures of education, age, gender and country of origin (in the case of Estonia, where about a third of the population is Russian origin); column 2 adds regional dummies (a dummy for the capital in the case of Estonia).

Our findings confirm the expectations of adverse effects of vocational education in re-entering employment. In both countries, each additional year of education raises the probability of re-entering employment by about 10 to 11%, while holding basic vocational or secondary vocational degrees reduces it by 12 to 16%. In the case of Poland, data allows us to control for the sector of the last job held by the currently unemployed worker, see column 3 in Table 2. The fact

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\(^{11}\) These distributions are found to provide the best fit for Estonia and Poland respectively, according to the Cox-Snell diagnostic plot. Results were virtually unchanged using the same distribution of the baseline hazard in both countries.
that vocational education is still significant means that within each sector workers with vocational education have lower probability of leaving unemployment than the average worker in the sector.12

All the results are robust to the inclusion of sectoral dummies, suggesting that our results are not driven by the fact that vocational workers are concentrated in particularly unfavorable sectors. Column 4 in Table 2 adds a tenure variable for Poland, defined as tenure in years in the last job of the unemployed worker. Unfortunately, this variable is not available in the panel of unemployed workers in Estonia. Tenure, controlling for age and education, has a significant negative effect on the hazard rate. This is consistent with our story, as workers with longer tenures on the previous job are likely to be endowed with more specific skills. Columns 5 and 6 in Table 2 and column 3 in Table 3 refer to the sub-sample of workers who declare to be unemployed due to dismissal from their previous job. An additional year of education raises the probability of finding a job by about 13 to 15%. This result is consistent with the view that, overall, the duration analysis is consistent with the view that, within each sector workers with vocational education have lower probability of leaving unemployment and, as expected, workers coming from the manufacturing sectors have a lower probability of exiting unemployment. However, there is no additional penalty from holding a vocational diploma and having previous work experience in the manufacturing sector.

reduce re-employment probabilities. Furthermore, dismissed workers with vocational education have additional difficulties to re-enter employment after job separation.

Table 2

<table>
<thead>
<tr>
<th>Model</th>
<th>Years of education</th>
<th>Secondary vocational</th>
<th>Basic vocational</th>
<th>Public job last</th>
<th>Age</th>
<th>Age2/100</th>
<th>Male</th>
<th>Married</th>
<th>Tenure</th>
<th>Constant</th>
<th>Time dum.</th>
<th>Region dum.</th>
<th>Previous sector</th>
<th>Dur. dependence</th>
<th>Unobs. heterog. (p-value)</th>
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</tbody>
</table>

Note: Absolute value of z-statistics in parenthesis. * and ** denote statistically significant at the 5 and 1% level respectively.

Table 3

<table>
<thead>
<tr>
<th>Model</th>
<th>Years of education</th>
<th>Secondary vocational</th>
<th>Basic vocational</th>
<th>Public job last</th>
<th>Age</th>
<th>Age2/100</th>
<th>Male</th>
<th>Married</th>
<th>Tullin (capital)</th>
<th>Constant</th>
<th>Time dum.</th>
<th>Dur. dependence</th>
<th>Unobs. heterogeneity (p-value)</th>
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</tbody>
</table>

Note: Absolute value of z-statistics in parenthesis. * and ** denote statistically significant at the 5 and 1% level respectively.

12 The results shown in column 3 include dummies for 16 sectors. We have interacted broad sector categories, i.e., manufacturing versus services, with the education variables in an attempt to identify if the wage penalty is larger when the worker with vocational education comes from a declining sector (in this case manufacturing). The results confirm that within each sector workers with vocational education have a lower probability of leaving unemployment and, as expected, workers coming from the manufacturing sectors have a lower probability of exiting unemployment. However, there is no additional penalty from holding a vocational diploma and having previous work experience in the manufacturing sector.
Early retirement, and more broadly movements out of the labor force become an option [sometimes unavoidable] for workers whose skills have become obsolete in a period of rapid structural change. There is evidence suggesting that this was an adjustment mechanism during the early transition in Poland. In 1991, soon after the fall of the iron curtain, the number of granted early retirement pensions in Poland reached 700,000. The conditions for receiving early retirement pensions were limited in 1997, but this increased the number of people who entered into pre-retirement programs. Between 1997 and 2002, more than 500,000 persons were granted such benefits. In this section we investigate the extent to which flows into inactivity among the older workers are an adjustment mechanism in Estonia and Poland, and whether workers with specific skills are more likely to become inactive once they are hit by a negative labor market shock.

We start the analysis computing all transitions between the three labor market states (employment, unemployment and inactivity) and report them in Table 6 [in Appendix B]. They are generally quite similar in magnitude in both countries, or slightly higher in the case of females in Poland, and whether workers with specific skills are more likely to become inactive in Estonia after 2002 (50–63). Females aged between 50 and 60, except in Estonia after 2002 (50–59). Marital status dummies (3 in Poland, 2 in Estonia) are also included in the regressions and found non-significant at standard levels.

### 3.4. Flows into inactivity

Early retirement, and more broadly movements out of the labor force become an option [sometimes unavoidable] for workers whose skills have become obsolete in a period of rapid structural change. There is evidence suggesting that this was an adjustment mechanism during the early transition in Poland. In 1991, soon after the fall of the iron curtain, the number of granted early retirement pensions in Poland reached 700,000. The conditions for receiving early retirement pensions were limited in 1997, but this increased the number of people who entered into pre-retirement programs. Between 1997 and 2002, more than 500,000 persons were granted such benefits. In this section we investigate the extent to which flows into inactivity among the older workers are an adjustment mechanism in Estonia and Poland, and whether workers with specific skills are more likely to become inactive once they are hit by a negative labor market shock.

We start the analysis computing all transitions between the three labor market states (employment, unemployment and inactivity) and report them in Table 6 in Appendix B. They are generally quite similar in magnitude in both countries, or slightly higher in the case of females in Poland, and whether workers with specific skills are more likely to become inactive once they are hit by a negative labor market shock.

The extent to which these workers who leave the labor force are those with specific skills is studied in Table 4. We show the marginal effects of probit estimates of the annual transition probabilities into those with specific skills.13 In alternative specifications we have allowed for a differential impact on the likelihood of moving into inactivity depending on the initial state (employment or unemployment). As expected, unemployed workers were more likely to move into inactivity. However, the positive effect of specific skills on the probability of becoming inactive did not differ significantly depending on the initial labor market state.

<table>
<thead>
<tr>
<th>Table 4</th>
<th>Flows into inactivity in the older population. Marginal effects from probit models.</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Poland</td>
</tr>
<tr>
<td></td>
<td>Males Females Males Females</td>
</tr>
<tr>
<td>Age</td>
<td>0.0022 0.0049 0.0029 0.0049</td>
</tr>
<tr>
<td>Years of education</td>
<td>−0.0017 −0.0028 −0.0031 −0.0027</td>
</tr>
<tr>
<td>Secondary vocational</td>
<td>0.0075 0.0052 0.0111 −0.0004</td>
</tr>
<tr>
<td>Basic vocational</td>
<td>0.0106 0.0102 0.0082 0.0090</td>
</tr>
<tr>
<td>County dummies Yes Yes Yes Yes</td>
<td></td>
</tr>
<tr>
<td>Quarter dummies Yes Yes Yes Yes</td>
<td></td>
</tr>
<tr>
<td>Observations</td>
<td>49192 34254 12129 10303</td>
</tr>
</tbody>
</table>

Note: Absolute value of z-statistics in parenthesis. * and ** denote statistically significant at the 5 and 1% level respectively. Males aged between 50 and 65, except in Estonia after 2002 (50–63). Females aged between 50 and 60, except in Estonia after 2002 (50–59). Marital status dummies (3 in Poland, 2 in Estonia) are also included in the regressions and found non-significant at standard levels.

#### 4. A dynamic continuous-time model

The empirical section delivered robust findings regarding the adverse effects of vocational education in a period of rapid reallocation. We now incorporate specific skills in a tractable macroeconomic model, in order to understand how they affect the dynamics: we will show that large amounts of specific skills dramatically slow down the adjustment of labor markets. In the next section we will extend this benchmark model to incorporate more realistic features of the Polish and Estonian labor market in order to match their recent unemployment experiences.

#### 4.1. Structure

Time is continuous. All agents are risk-neutral and discount future at rate r. There are three sectors of production in the economy, two intermediate sectors and a final good sector. The first intermediate sector is the traditional sector (say, typically traditional industry or agriculture), and it is denoted by subscript o standing for old. The second one is a modern sector (say, services or high-value added industry), denoted by the subscript n standing for new. Each sector produces Yo and Yn, respectively. The production technology for the final good is Y = (aoYo + anYn)1/ρ with ao + an = 1. This structure closely corresponds to Acemoglu (2001). In each sector, production is sold in competitive markets, so that, denoting their price by pk (for k = o,n) we have

\[ p_k = a_k Y_k^{1-\gamma} Y^{\gamma-1} \quad \text{for} \quad k = n, o. \]  

(1)

Firms in the intermediate sectors require labor. We follow the “small firm” assumption in Pissarides (2000), that is: a firm only requires one worker to produce. We detail the structure and environment of these firms later on. New firms, in the tradition of the matching literature, post a vacancy at a flow cost γk and recruit workers from the labor market. As expected, unemployed workers were more likely to move into inactivity. However, the positive effect of specific skills on the probability of becoming inactive did not differ significantly depending on the initial labor market state.
randomly, according to another Poisson process denoted by \( q_k(t) \). If we denote by \( e_k \) the number of jobs in each intermediate sector, normalizing productivity to 1, we have that \( V_k = e_k \).

We denote by \( l_k \) the labor force in each sector and by \( u_k = l_k - e_k \) the number of unemployed workers. Match formation occurs through some perfectly segmented matching process: each unemployed worker cannot apply to more than one job. The per unit of time number of matches \( M_k \) is given by \( M_k = h(u_k, V_k) \) for \( k = n, o \), where \( V_k \) is the number of vacancies posted. The function \( h \) is assumed to have aggregate constant returns to scale and decreasing returns to scale in each argument. Furthermore, partial derivatives \( \partial h/\partial u_k \) and \( \partial h/\partial V_k \) tend to infinity in zero and to zero in infinity. Denoting by \( \theta_k = V_k/u_k \) the sectoral tightness of the labor market, we have that \( q_k(t) = h(u_k, V_k)/V_k \) for \( k = n, o \) and \( \phi_k(t) = h(u_k, V_k)/u_k = h\theta_k(t) \) for \( k = n, o \), where \( \phi_k \) is the rate at which workers find a job, \( q_k \) the rate at which vacancies are filled in and \( \phi_k < 0 \) and \( \phi_k > 0 \). Population of workers is normalized to 1, with a fraction \( \delta \) newly born, and an equivalent mass that exogenously disappears from the labor force per unit of time. Later on we allow for other sources of exiting the labor force such as early retirement.

### 4.1. Labor supply and sectoral allocation of workers

Labor supply depends on the allocation of skills. It has three margins: initial education, mobility across sectors through retraining, and early retirement. In this section we develop the core model with education, and we will extend it by allowing for retraining and early retirement in Section 5. To simplify the theoretical analysis, we think of labor supply in each sector as being determined by the type of initial education of individuals. Each worker is thus assigned to a sector. We assume that education is instantaneous and costless, and provides skills with certainty. It is important to note at this stage that, given their nested structure, sectors could equivalently be reinterpreted as occupations. Workers’ imperfect mobility across sectors would then be reinterpreted as imperfect mobility across occupations.

General skills, which are provided by general education, are required to work in the modern sector. In order to work in the traditional sector specific skills are sufficient, and they are provided by vocational education. Consistently with the traditional view of specific and general skills, the portability of skills is asymmetric: general skills can be used in the traditional sector (i.e., in managerial occupations for instance); while vocational education cannot be used in the modern sector. However, in order to simplify the derivation of Bellman equations we assume that workers with general skills do not apply to jobs in the traditional sectors, since this will never be in their interest. We propose in Appendix A.8 a condition for this proposition to apply.

Denote by \( \nu(t) \) and \( 1 - \nu(t) \) the share of a cohort of workers born at time \( t \) attending vocational and general education respectively, whereby \( \nu \) is controlled by a central authority (government) fixing the allocation of schools. This quantity is predetermined, i.e., at a given point in time a cohort of workers of age \( a \) was trained in proportions \( \nu(t)\left(t - a\right) \) and \( 1 - \nu(t)\left(t - a\right) \). Adjustments of the labor supply of workers through initial education thus only occur at the margin, with newborn workers. In the long-run education is endogenous: the government adjusts \( \nu \) so that it is determined according to the free-entry of workers in each sector. The endogenous determination of \( \nu \) is discussed later on. We denote by \( \nu^e \) the equilibrium, long-run value of \( \nu \). Fig. 2 describes the individual trajectories of workers in the life-cycle.

#### 4.1.2. Firms in the intermediate good sector

As already mentioned above, firms in the intermediate good sector produce with only one worker. Denote by \( V_k \) the time-varying asset value of a job vacancy and by \( J_k \) the time-varying asset value of a filled job. Following Pissarides (2000) and Mortensen and Pissarides (1999), we assume that firms enter and exit freely at the vacancy posting stage: this implies that \( V_k(t) \equiv 0 \). If this equality were not satisfied, there would be either unexploited profit opportunities (if \( V_k \) was strictly positive) or expected losses from a vacancy (if \( V_k \) was strictly negative). The important implication is that the supply of vacancies adjusts instantaneously, even along the transition paths after an aggregate shock. We have all arbitrage equations for \( k = n, o \) in Appendix A.1, and focus on the derivation of the steady-state and out of steady-state equations here. Note that although we present the equations in a simple way, their derivation out of steady-state is not straightforward and implies several steps derived in Appendix A.

Firms having recruited a worker can start to produce. Their revenue is a function of the price of the good \( p_k \), the wage of their worker \( w_k \), and some operating costs denoted by \( \Omega \). All three variables potentially depend on time \( t \). Further, firms face idiosyncratic shocks affecting their revenue function. We assume that these shocks affect the value of the operating cost and occur with Poisson intensity \( \lambda \). Initially, at the time of job creation, \( \Omega \) is zero. Then, it takes a random value at each shock, the new value being drawn from a distribution with density \( g \) and cumulated density \( G \), on a support \((0, \Omega^*)\). When \( \Omega \) grows too large, the job may be destroyed.\(^{16}\) In addition, we assume that matches are destroyed when workers leave the labor force (rate \( \delta \)), and there is also a purely exogenous destruction component denoted by \( \Xi \) featuring any shock unrelated to operating costs \( \Omega \).

\(^{16}\) We assume a shock on operating cost and not on productivity so as to avoid aggregate sectoral prices to be affected by the idiosyncratic productivity of firms. In such a case prices in each sector would depend in a complicated way on the cross-section of surviving firms, and thus on the job destruction rule, without adding additional insights for the problem we analyze here.
4.2. Equilibrium

The value of $R_k(t)$ is such that $J_k(R_k(t),t) = 0$. Using this equality together with Eqs. (A2) and (A4), we obtain the dynamic job destruction condition:

$$R_k(t) = p_k(t) - w_k + \frac{\lambda k(t)}{r + \lambda + \delta + \sigma_k} \frac{\partial J_k(R_k, t)}{\partial t}.$$  \hspace{1cm} (2)

This equation determines a positive relation between the level of profits and the reservation operating cost, which happens to be independent of labor market tightness. Straightforward differentiation shows that, at constant $\partial J_k/\partial t$, the higher the revenue of the firm ($p_k - w_k$), the higher $R_k$, i.e., the higher the operating cost the firm can cope with without closing down. Note also that out of the steady-state a positive change in the value of the job raises $R_k$: when the value of a job for the firm appreciates, the firm is more reluctant to close down the margin. The endogenous component of the destruction rate is $\lambda[1 - G(R_k(t))].$ The total job destruction rate faced by firms is $\delta + \sigma_k + \lambda[1 - G(R_k(t))]$ and is denoted by $\partial J_k(t)$.

Similarly, a dynamic job creation condition can be derived from Eqs. (A1) and (A2) in Appendix A.1:

$$\frac{\gamma_k}{q(J_k(t))} = \max_{0 \leq J_k(0,t)} \frac{R_k(t)}{r + \lambda + \delta + \sigma_k},$$  \hspace{1cm} (3)

This equation states that the expected value of search cost $\gamma_k/q(J_k(t))$ has to equal the present-discounted value of profits to the firm, taking into account the turnover rate of workers. The Max operator simply makes sure that when profits from new jobs go negative, firms stop creating them and the vacancy rate is equal to zero; this can occur along the dynamic paths but it is not so much of an issue in steady-states. This equation delivers a positive relation between labor market tightness and $R_k$, which simply states that the longer the expected duration of a job, the larger job creation rate. At a fixed $R_k$, this also delivers a positive link between $\theta_k$ and $p_k$: the higher the demand for a sector, the higher job creation.

We finally need an equation for wages. Recently, Hall (2005) and Shimer (2005) have argued that the dynamic properties of matching models are more accurate with rigid wages. We will also derive the benchmark properties of the model with rigid wages, but for a slightly more general specification in postulating a static rule of wages such as

$$w_k = w[1 - \beta] + \beta p_k,$$  \hspace{1cm} (4)

where $\beta$ captures the extent to which wages reflect the marginal product of workers. When $\beta = 0$ wages are fixed, thus totally rigid. When $\beta = 1$ the wage equals the marginal product. If the marginal product changes in time, so does the wage. Note that this wage structure will lead, in some cases, to the destruction of viable jobs, i.e., jobs associated with a positive surplus. Wages are too rigid here to allow for a wage drop, implying some inefficient job destruction.\footnote{Indeed, it is possible to show that there exists a wage, say $\omega$ strictly below $w_k$, for which $J_k(\Omega,t,\omega)$ remains positive while at the same time workers prefer employment to unemployment; i.e., $\omega$ is greater than workers’ reservation wage $(r + \delta)U_0$.}

We find our assumption of inefficient destructions more appealing along the process of reallocation of employment across sectors due to structural changes, as clearly workers in declining sectors are not ready to work at any wage. The simplest rationalization is that, in sectors covered by minimum wages or collective wage setting, workers’ unions may exert pressure to avoid downward wage bidding.

4.3. Dynamic path

4.3.1. The shock

We now use Eqs. (2) and (3) to simulate the path of adjustment following a sectoral shock. The experiment we run is the following: we start at time $t_0$ from a long-run steady-state in which unemployment is at some benchmark value, say 10%, and the demand for both goods is identical: $a_0 = a_1 = 1/2$. All sectoral parameters are also assumed to be identical, notably sectoral wages and job destruction rates. This implies that the endogenous supply of education is identical across sectors: half of the newborn workers go to vocational education, the other half goes to general education ($\nu(t_0) = 0.5$).

We then let at time $t_0$ the demand for the goods of the new sector increase relative to the demand for the old sector, featuring either an opening of the country to international trade as a consequence of the enlargement, or the adjustment of production to biased technical progress or shifts in demand. As a benchmark, we assume that $a_0$ is raised to 0.64 and $a_1$ falls to 0.36. As $a_0$ changes relative to $a_1$, the aggregate price index may change as well. So we decide to divide the price of each good by the conventional price index being $P = (a_0 p_0^{\alpha_0} + a_1 p_1^{\alpha_1} - \beta_0) / (\alpha_0 - \beta_0)$. In all experiments we do, in order to avoid this distortion—the marginal change in the level of prices affects the aggregate demand for jobs independently of the distributional effects we want to underline. The initial value of real prices $p_k/P$ in the symmetric steady-state is 1 in each sector.

4.3.2. Steady-states

In each steady-state, the following holds. First, all derivatives with respect to time are equal to zero in Eqs. (A10)–(A12) presented in Appendix A.4. Labor market tightness $\theta_k$ is determined through Eq. (3). Once $\theta_k$ is known, we know the rate of access to jobs $\phi_k$ and thus $R_k$ and $J_k$. Denoting by a star the steady-state level of a variable, we have:

$$u_k^* / \theta_k^* = \frac{J_k^*}{\phi_k^* + J_k^*}; e_k^* / \phi_k^* = \frac{\phi_k^*}{\phi_k^* + J_k^*}.$$  \hspace{1cm} (5)

In the initial steady-state, $\nu$ is at its initial value ($\nu^* = 0.5$) if the economy has two perfectly symmetrical sectors. In this equilibrium, as we can see from Eq. (3) $\theta_k$ depends on the price of each good $p_k$. Note also that from the price Eq. (1), sectoral prices are linked to $Y_0$ and $Y_n$, while those quantities are themselves linked to $\theta_k$ by the employment equations presented in Eqs. (A15) and (A16). Overall, we have here eight equations and eight unknowns ($R_0, \theta_0, p_0$ and $Y_k$) for $k = o, n$.

4.3.3. Transition between steady-states

To obtain the dynamics of employment and unemployment, one has to take care of an additional complication due to the fact that $R_k$ may jump from time to time (in our case, only at the time of the shock to $a_0$). A discontinuous decrease in $R_k$ leads to a mass of job destruction, by a quantity $\Sigma_k = \phi_k[R_k - G(R_k)]$ if $R^* < R$ and 0 otherwise, where $R^*$ and $R$ represent the value of $R_k$ after and before the jump. In fact, in the flows equations $\Sigma_k$ has to be multiplied by a Dirac function (denoted by $\Delta(t_0)$) defined at the time of the discontinuous decrease of $R_k$.\footnote{A Dirac is the equivalent of a mass point in time, i.e. it is a distribution defined by its integral over an interval: the integral is equal to 1 if the interval of integration encompasses $t_0$.} In our case, employment and unemployment in each sector evolve according to three differentials equations presented in Appendix A.2: Eqs. (A7)–(A9).
We need then to investigate the dynamics of $\theta_t(t)$ and $R_k(t)$. During this transition, we assume as in Pissarides (2000) that the free-entry condition in each sector is always satisfied. Note however an important difference with the “traditional” dynamics in Pissarides, where $\theta$ is time invariant after a shock because it immediately jumps to its new steady-state value. In our case, $\theta_t$ depends on profits and thus on prices of the good in each sector. The latter varies slowly over time, because the price is the marginal product, which depends on the production through the stock of employees in each sector. Employment is a state variable, thus time varying; at each point in time agents create the relevant amount of vacancies consistent with free entry. However, we retain from the traditional analysis that the convergence of agents towards the current zero-profit value of $\theta_t$ is instantly fast.

Finally, in investigating the transition dynamics we need to determine the evolution of the supply of education over time. Here, we assume that the policy maker is well informed or alternatively that the education choices are made by individuals and so the fraction of students enrolled in vocational education is the long-run target.\(^{19}\)

$$\nu(t) = \nu^*, \quad \text{(6)}$$

where $\nu^*$ is the steady-state value after the shock. This constitutes a ninth unknown, and we consequently require an additional condition to solve for its value. A benevolent government would choose $\nu^*$ so as to equalize the value of starting in each sector, i.e. in this case $\nu^*$ would be determined by the equality of the present-discounted value of unemployment, i.e. $\Gamma_0 = \Gamma^*$. If $\sigma^*_t = \sigma^*$ and $\beta = 0$ (exogenous wages), this equality takes a particularly simple form, as it implies the equality of labor market tightness across sectors: $\theta_t = \theta_o$.

4.3.4. Linearization around the steady-state and saddle-path properties

We consider $t-t_0$ (so that $\Sigma_0 = 0$), and interior solutions with positive tightness, and discuss other solutions in Appendix A.7. In matching models, the dynamics is usually described by a saddle-path, as the vacancy opening decision is forward-looking and costless, so that $V$ and thus $\theta$ can jump instantaneously. In Pissarides (2000) for instance, this is shown by linearizing the dynamic system in $(u, \theta)$ around the equilibrium steady-state and recovering a positive and a negative eigenvalue of the corresponding matrix. We can proceed in a similar way here, with the difference that there are two sectors, and in each sector labor supply is not constant, so that our system is eight-by-eight. However, given our assumptions the system is block diagonal, which allows one to focus on a four-by-four subsystem for each sector.

Using Eq. (3), we see that the cut-off cost $R_k$ and labor market tightness move together along the equilibrium path: this equation generates a positively sloped relation between the two endogenous variables. This implies that the dynamics of $\theta$ and $R$ is exactly the same. One can also easily log-linearize the dynamic equations for employment and unemployment. Denoting by $\Lambda = (r + \lambda + \hat{\sigma} + \bar{\sigma}) > 0$, we obtain a block diagonal matrix represented in Appendix A.6 (Eq. (A19)). This matrix has four eigenvalues denoted by $\lambda_1$ to $\lambda_4$ with $\lambda_1, \lambda_3 > \Lambda > 0$ and $\lambda_2 > \Lambda$. To see the latter point, the determinant of the upper-left $2 \times 2$ block in the matrix is $\lambda_1 \lambda_2 - \hat{\sigma} (\hat{\sigma}^* + \bar{\sigma}^*) > 0$ while the trace is $(\lambda_1 + \lambda_2) > 0$, indicating that both $\lambda_1$ and $\lambda_2$ are necessarily negative. Thus, we have in each subsystem two variables exhibiting stable dynamics and two exhibiting explosive dynamics. We show in the Appendix A.5 that the dynamic evolution of the system around the steady-state is thus described by a unique generalized saddle-point with four forward-looking variables, $\theta_0(t)$ and $R_k(t)$ and four state-dependent variables (employment and unemployment in each sector), plus three variables implied by these dynamics ($p_t(t)$ for each sector and $\nu(t)$). It follows that:

**Proposition 1.** The dynamics of transition is a saddle-path, agents coordinating spontaneously so that forward-looking variables converge immediately on this saddle-path to finally converge to the steady-state where $\theta_t(t) = \theta^*_t$ and $R_k(t) = R^*_k$. All predetermined variables are continuous when $t \to t_0$.

Appendix A.7 contains several comments on technical aspects of the dynamics.

### 4.4. Numerical solutions

#### 4.4.1. Parameter determination

This section illustrates the dynamics of an economy with rigid wages and specific skills. Relevant extensions, notably flexible wages, retraining and early retirement policies, unemployment benefits and employment protection are postponed to the next section in order to match the Polish and Estonian cases. We fix the parameters to the initial steady-state levels so as to have a symmetric equilibrium across sectors. This means that the aggregation function of intermediate goods into the final good has equal shares, namely $a_0 = a_1 = 1/2$. The demographic parameter $\delta$ is a crucial quantity for the speed of adjustment of the pool of skills. We set it to $\delta = 0.00625$ per quarter, so the average working life of individuals is 160 quarters, i.e. 40 years. The discount rate is $r = 1\%$ per quarter. Initially, the education parameter $\nu$, i.e. the share of workers in vocational education, is at its equilibrium value $\nu^* = 0.5$. Other parameters are set at values insuring that the unemployment rate is around 10%, thus employment in each sector is 0.45. The fixed part in wages (that is, the value of unemployment benefits) is two thirds of total productivity.

The job destruction rate is slightly below 3.3% per quarter, and tightness of the labor market is initially around 1.8 in each sector. Taking into account the parameters of the matching function (scale and elasticity) this implies an average unemployment duration of 8.7 months.\(^{20}\)

#### 4.4.2. Simulation

We now shock the relative demand for good $n$ by increasing $a_n$ from 0.5 to 0.64 and reducing $a_0$ from 0.5 to 0.36. Hence, the relative demand index $a_n/a_0$ passes from 1 to 1.78. To compute the transition path, we have used a standard numerical tool discretizing time intervals in order to approximate the solution of ordinary non-linear equations.\(^{21}\) Fig. 3 illustrates the dynamics. The x-axis on all figures is time elapsed since the initial jump of $a_n$ (resp. $a_0$) from 0.5 to 0.64 (resp. to 0.36). Time units are quarters. At $t = +\infty$, the system converges to a new equilibrium steady-state. Unreported simulations for very large time intervals actually show the perfect fit between this limit and the final point we would expect from the steady-state equations when $a_n/a_0 = 1.78$: unemployment converges to the initial level at 10% of the labor force.

As the left graph in Fig. 3 shows, unemployment in the expanding sector declines, while it reaches very high levels in the old sector. Indeed, at the time of the shock, a mass 0.075 of workers is displaced in the old sector, which reaches an unemployment rate above 25%.

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\(^{19}\) In the discussion paper version of this article, we studied a fairly more general case $v(t) = 1 + (\nu - v^*)(1 - e^{-\alpha t})$, where $\nu = 0.5$ is the initial symmetric equilibrium value of $v$ and $\alpha$ is the speed of convergence, chosen to be infinity here. The choice of $\alpha$ did not make a substantial quantitative difference except for $\alpha = 0$ (permanent mismatch).

\(^{20}\) Other parameters are: scale parameter in matching $A = 0.257$; matching elasticity of unemployment $\gamma = 0.5$; complementarity parameter in production $\rho = -1$; exogenous job destruction $\Pi = 0.1$; upper support of idiosyncratic shocks $\Omega^* = 0.8$; frequency of idiosyncratic shocks $\lambda = 0.045$; hiring costs $\gamma_{u} = 1.05; \gamma_{u} - 2$.

\(^{21}\) All simulations were made with ode23 or ode45 in Matlab R, version 7.0.4, which compute the solution to a system of non-linear differential equations. The use of alternative Matlab algorithms did not make any difference in the dynamic paths. All our codes are available upon request.
This raises total unemployment to 17.5% of the labor force. The convergence in the unemployment rate is first rather fast, as total unemployment falls to 14% in less than 10 quarters. However, during a second phase the convergence back to the 10% steady-state level is extremely slow: 40 quarters (10 years) after the shock, the level of unemployment is 2.5 percentage points above its long-run value. This is because unemployment in the old sector is still 7.5 percentage points above its long-run value during the first 15 years after the shock.

This slow adjustment might seem surprising taking into account that education adjusts immediately to the new steady-state level after the shock. However, education only affects the new entrants in the labor market, leaving aside the stock of older workers. This results in a massive long-lasting level of mismatch in the economy.

The evolution of the job destruction rates by sector explains these dynamics fairly well: at the time of the shock, there is a jump corresponding to the mass of 0.075 units of workers being displaced, and post-jump job destruction rate in the old sector rises and reaches a peak at 4.3%, much above its long-run level (3.27%). At the same time, job destruction in the new sector falls to 2.5% and only gradually rises again. Aggregate output declines strongly, by the log of 0.71/0.55 or 25%, and gradually recovers after 2 to 3 years. Fig. 3 in Appendix B reports the evolution of additional variables. Employment in the old sector first falls drastically, as there are many obsolete firms instantaneously destroyed. Then, given the excess supply of workers due to these immediate layoffs, employment starts rising reaching it long run level fairly fast. The increase of employment in the good sector is slower, due to market frictions and even more importantly a lack of labor supply.

To summarize our results, we can distinguish three distinct phases in the unemployment dynamics after the asymmetric shock shown in Fig. 3. Using Eq. (A19), one can in fact characterize the speed of convergence of each of these phases.

**Proposition 2.** The speed of convergence after an asymmetric shock presents three phases:

1. Using the saddle-path property ($\Lambda>0$), there is a first period characterized by a large and instantaneous rise in unemployment, following the immediate death of a large mass of firms in the declining sector.
2. There is a second period of rapid adjustment, where firms facing a large pool of unemployed workers post more vacancies. The speed of adjustment here is dominated by the eigenvalue with the largest absolute value ($\lambda_1$).
3. There is an additional horizon of very slow convergence. The speed of adjustment now is dominated by $\lambda_2$, which has the lowest absolute value.

To give a feeling of the relative value of $\lambda_1$ and $\lambda_2$, one can make the following approximation: if $|\lambda_2| \ll |\lambda_1|$, then $\lambda_1 \approx - (\delta + \beta D^k + \phi^k) = -0.3765$ while $\lambda_2 = -\delta \frac{\beta D^k + \phi^k}{\beta + \delta + \phi^k} = -0.0066 - \delta$. Note that $\lambda_2/\lambda_1 \approx 0.01$ so that the approximation is valid. In other words, the long-
run convergence to the steady-state is governed by the slowest adjustment mechanism, namely by the demographic turnover. This is the necessary time for older workers with inadequate skills to have retired and be replaced by a labor supply with the right mix of skills. Until this happens, mismatch between demand and supply of skills across sectors persists.

Finally, given the crucial role of the speed of demographic transition in the model we have run experiments varying $\delta$ in the simulations. The relationship between the speed of transition and $\delta$ is log-linear, hence the dynamics are virtually unaffected even if we reduce the average working life substantially: e.g., to 25 years. However, further increases in $\delta$ have a sizable impact. Raising the demographic turnover from $\delta = 0.00625$ to 0.015 (which implies an average working life of 12.5 years) and adjusting wages to 0.587 in order to keep the same steady-state unemployment rate as in the benchmark case yields a convergence rate almost three times faster: it takes only 20 quarters to reach an unemployment rate of 17.5% in the old sector, instead of 50 in the benchmark. The next section introduces additional flexibility into the model in an attempt to match the observed dynamics of Estonia and Poland and provide additional robustness exercises.

5. A tale of two countries?

We now return to the motivation, and provide a quantitative account of the impact of the different institutional settings in Estonia and Poland. As briefly outlined above, Poland is characterized by relatively high wage rigidity, high unemployment benefits, stringent firing costs and generous early retirement programs, while Estonia presents very flexible labor markets with an emphasis on active labor market policies. We therefore extended the model of the previous section to include government-sponsored retraining and early retirement policies as well as firing costs, in an attempt to match the data in the two countries. All derivations of the extended model are presented in Appendix A.8.

A first target to match is the level of steady-state unemployment, around 10% in both economies. A second statistic is the expected duration of unemployment. To do this, given right-censoring in the data, we consider the median duration in unemployment in both countries. The median duration, denoted by MD, is 8 months in Estonia and 14 months in Poland. Given that the job finding rate $\phi$ is also a Poisson intensity, this means that the distribution of completed spells is $e^{-\phi t}$, implying a quarterly job finding rate that should be matched equal to $\phi = \ln 2/(MD/4)$, or $\phi = 0.347$ in Estonia and $\phi = 0.198$ in Poland. These two statistics are matched in setting firing costs to zero ($F = 0$) in Estonia and $F = 2.5$ or approximately 2 quarters of production in Poland. Setting hiring costs such as $\gamma_k = 1.05 + F^2 \times 0.385$ roughly insures that the steady-state value of unemployment is equalized in the two countries at 10%. In the absence of known data on wage flexibility, we set the wage to $2/3$ in Poland and to $1/3 + 1/3 p_k$ in Estonia, which captures higher wage rigidity in Poland and fixes the initial wage to be the same in both countries. Moreover, and consistently with the institutional features of both countries, this implies that the value of unemployment benefits is higher in Poland than in Estonia.

23 See Lamo et al. (2006) for further background information for the two countries.
24 Given that along the transition, $\phi_o$ and $\phi_n$ appear to be relatively symmetrical around the steady-state level $\phi_o^* = \phi_n^*$, we can match the steady-state level of $\phi^*$ with the number implied by the data, even though the data does not represent the steady-state.
The third statistic matched is the initial value of the relative demand for the old sector (parameter $a_o$), corresponding to the share observed in the data of vocational workers in the steady-state: 0.66 in Poland and 0.34 in Estonia. We will impose the same relative shock on each economy, i.e. the final value of $a_o$ is 33% lower in each country, so as to obtain an initial peak in unemployment which is roughly equivalent to that observed in the data. Fourth, we attempt to match two additional labor market policies that differ markedly across these two countries: the fraction of the unemployed eligible for retraining ($\tau$) and early retirement ($\pi$) policies. According to the 2003 Report from the Ministry of Economy, Labor and Social Policy in Poland, there were 47,600 workers under training, i.e. approximately 0.18% of the total labor force in 2001. Hence, we set $\tau = 0$ in Poland. There were at the same time 479,100 pre-retirement allowances and pre-retirement benefits, or 1.84% of the labor force. Setting $\pi = 0.05$ in the calibration between quarters 4 and 44 implies that, 5 years after the policy has been set, the fraction of early-retirees is 1.8% of the labor force, with a maximum of 2% in the sample. In Estonia, there is virtually no early retirement policy, so that $\pi = 0$, whereas according to Eamets et al. (1999) about 10% of the total pool of the unemployed receives some training. Training in the model is instantaneous so the comparison with actual figures is impossible. Hence, we set $\tau = 0.05$ to be symmetrical with the early retirement policy in Poland and see whether the dynamics of unemployment in the calibration matches the Estonian labor market. Other parameters are the same as in the benchmark calibration.

Table 5 and Figs. 5 and 6 provide an account of how a combination of initial conditions, labor market institutions and shocks generate the type of divergence in unemployment dynamics observed in the data. With regards to unemployment stocks, the data are relatively well replicated in the simulations: unemployment peaks at 15% in Estonia, and slightly above 21% in Poland, as compared to 13.5% and 20% respectively in the data. The figures clearly show the difference in unemployment persistence between both countries. Ten years (40 quarters) after the shock, unemployment is still above 13% in Poland, while it has almost converged to its steady-state value of 10% in Estonia. Moreover, while the unemployment gap between workers with vocational skills and workers with general education has rapidly closed in Estonia (less...
than 3 percentage points 10 years after the shock) it remains above 12 percentage points in Poland. This is consistent with the evidence previously discussed, where we showed a stronger negative impact of vocational skills in the likelihood of exiting unemployment in the Polish economy. The fit is also reasonably good in terms of unemployment spells for Estonia (implied median duration is 8.2 months instead of 8 in the data), but less good in Poland where the implied median spell is 15.6 months according to the model, 1.6 months above the data. However, given that measurement of unemployment spells is quite imprecise, we have not tried to improve over these statistics.

The next and natural question is to assess the contribution of institutions, policies and skill specificities in the differences between both countries regarding unemployment dynamics. According to unreported counter-factual simulation exercises, the relative importance of the various factors is as follows: if we impose the same absolute shock instead of the same relative shock to both economies, approximately half of the difference in unemployment persistence disappears. Having the same absolute shock means that initial conditions as regards the pools of different types of educational diplomas are identical, so that we can conclude that half of the diverging evolution between the two economies is due to different stocks of skills in the population, with too much vocational skills in Poland. The other half is due to differences in labor market institutions. Imposing $t=0$ in Estonia implies that after 5 years unemployment is around 12% instead of 11%, meaning that retraining could account for one quarter to one third of the difference with Poland. The remainder can be attributed to higher wage rigidity, more generous unemployment benefits and stringent dismissal laws in the Polish economy.

Two disclaimers apply to our attempts to match the facts with our theoretical framework. First, the decomposition carried out is not easy to do, as each factor interact with the others and a proper decomposition should also account for those interaction terms, which we cannot easily do given the non-linear nature of the system we explore. Second, some mechanisms of potential interest have not been included in the model: job search effort is not endogenous and in reality unemployment compensation differs between the two countries, constituting an additional source of divergence in persistence. The potentially large costs of early retirement policies and retraining have been also left out of our analysis. For instance, Malthusian policies—that is, adjustment by downsizing the labor force through early retirement—as in Poland might be efficient at reducing unemployment in the absence of funding issues, but certainly raise the tax burden imposed to the economy. These issues are left to future research.

6. Concluding comments

Our paper has emphasized the role of specific skills on the (lack of) speed of adjustment of labor markets in periods of rapid structural change. Those episodes are frequent, possibly increasingly frequent with trade integration (agreements such as NAFTA and the successive enlargement rounds of the European Union) and are likely to bring about an intense process of sectoral restructuring, which usually requires significant reallocation of workers across firms and industries. Technological changes that rapidly alter the productive structure of firms are another example where significant hiring and firing of workers will coexist. When workers have adaptable skills these processes are smooth, and the gains from trade liberalization and technological change are rapidly realized. However, workers’ skills are often not ready for the new economic environment, and retraining might be too costly and lengthy. In this case, shortages of workers with adequate skills might become a long-lasting phenomenon, and the structural reallocation might be coupled with high and persistent unemployment.

Appendix A. Model

A.1. Bellman equations for firms

The value of a vacancy is given by

$$rV_k = -\gamma_k + q_k(k-V_k) + \frac{\partial V_k}{\partial t},$$

(A1)

stating that firms realize a capital gain $J_k - V_k$ at the time of recruitment and take into account their flow costs and the possible change in the value of vacancies in time. As discussed in the text, $V_k$ is equal to zero at all times. Hence, we have that

$$rJ_k(\Omega, t) = p_k - w_k - \lambda J_k(\Omega, t) - \int_0^T \max(J_k(\Omega, t), 0)dG(\Omega') - J_k(\Omega, t)$$

$$- (\delta + \sum\nu)J_k(\Omega, t) + \frac{\partial J_k}{\partial t}.$$  

(A2)

This equation states that the equity value of the firm is the firm’s profit, plus firm’s anticipation of a capital loss $V_k - J_k(\Omega)$ due to exogenous separation from its worker, and of a capital change $J_k(\Omega, t) - J_k(\Omega, t)$ when $\Omega$ changes. However, the firm retains the option of firing the worker if the new value $\Omega$ is too large, hence the Max operator. It finally takes into account the non-stationarity of its environment through the partial derivative term. Differentiating this equation with respect to $\Omega$, we obtain that

$$(\rho + \delta + \lambda + \sum\nu)\frac{\partial J_k}{\partial \Omega} = -1 + \frac{\partial^2 J_k}{\partial \Omega^2}.$$  

We restrict the solutions for $J_k$ to those cases in which the dependence of $J$ on $\Omega$ is time invariant, i.e.25

$$\frac{\partial^2 J_k}{\partial t^2} = 0.$$  

(A3)

Given that the slope of $J_k$ is constant when $\Omega$ is below some value $R_k$, we can rewrite without loss of generality the value of a job as:

$$J_k(\Omega, t) = \frac{R_k(t) - \Omega}{\rho + \delta + \lambda + \sum\nu}.$$  

(A4)

A.2. Stock-flows equations

Flows in and out each skill level are governed by

$$\frac{\partial l_k}{\partial t} = \delta v - \delta n - T - P,$$  

(A5)

$$\frac{\partial l_k}{\partial t} = \delta (1-v) - \delta n + T,$$  

(A6)

where $l_k$ is the labor force in sector $k$, with $l_k = e_k + u_k$, and where $T$ and $P$ are flows into retraining and early retirement as defined in Appendix A.8. In the benchmark model, they are set to zero. However, to minimize the length of exposition, we will keep them here so as to avoid repeating the augmented equations later on. Similarly, the dynamic adjustment of employment and unemployment is now described by the following system:

$$\frac{\partial \phi_k}{\partial t} = \phi_k u_k - J \phi_k(\Omega)e_k - \sum\nu \Delta(t_0)$$  

(A7)

$$\frac{\partial \phi_k}{\partial t} = \delta v + [J \phi_k(\Omega) - \delta \nu e_k - T(t) - P(t)] - (\delta + \phi_k)u_k + \sum\nu \Delta(t_0).$$  

(A8)

$$\frac{\partial u_k}{\partial t} = \delta (1-v) + T(t) + [J u_k(\Omega) - \delta \nu e_k - (\delta + \phi_k)u_k].$$  

(A9)

25 As we shall show, these solutions exist. We have not explored the other solutions in which this dependence varies with time.
where \( J_{D_\kappa}(t) = \sqrt{\epsilon_\kappa} + \lambda[1 - G(R_k(t))] + \delta \) is the job destruction rate faced by firms.

### A.3. Bellman equations for workers

The value of employment and unemployment is defined by:

\[
(r + \delta)U_k = b + \phi_n(W_k - U_k) + \partial U_k / \partial t, \quad \text{for} \quad k = n, o, \quad (A10)
\]

\[
(r + \delta)W_n = w_n + (J_{D_\kappa} - \delta)(1 - \tau - \pi)U_o + \tau(U_n - C) - W_n + \Pi, \quad \text{and} \quad \partial W_n / \partial t, \quad (A11)
\]

\[
(r + \delta)W_o = w_o + (J_{D_\kappa} - \delta)(U_n - W_n) + \partial W_n / \partial t. \quad (A12)
\]

where \( b \) is the flow utility from unemployment. As above, we already introduce notations \( \Pi, \pi \) and \( \tau \) defined in Appendix A8, which are respectively utility derived from government-sponsored pre-retirement, a Poisson transition rate of displaced workers of the old sector into pre-retirement and a Poisson transition rate of displaced workers of the old sector into a retraining scheme. For the benchmark model, simply set \( \pi = \tau = 0 \).

### A.4. Steady-states

Indexing steady-state variables with a superscript * and using Eqs. (A5) and (A6), we obtain:

\[
l^n* = v^* - (T + P) / \delta, \quad (A13)
\]

\[
l^o* = 1 - v^* + T / \delta. \quad (A14)
\]

In steady-state, \( l^n* + l^o* = 1 - \pi / \delta \), where \( P \) and \( T \) are defined above \( (P = T = 0 \) corresponds to the benchmark model). Using \( u_k = l^n_k - e_k \) into Eq. (A7), we obtain the steady-state employment and unemployment rates as in Eq. (5) and finally using Eqs. (A13) and (A14) into Eq. (5), we obtain the level of employment and thus of production of each good:

\[
e^n* = (1 - v^* + T / \delta) = \frac{\phi_n'(\theta_n)}{J_{D^n} + \phi_n(\theta_n)} = Y_n*, \quad (A15)
\]

\[
e^o* = (v^* - (T + P) / \delta) = \frac{\phi_o'(\theta_o)}{J_{D^n} + \phi_o(\theta_o)} = Y_o*. \quad (A16)
\]

### A.5. Dynamics

This Appendix presents five Lemmas used in the derivation of the dynamic properties of the model.

**Lemma 1.**

\[
\frac{\partial j_k(0,t)}{\partial t} = \frac{\partial \theta_k}{\partial t} \cdot \eta \cdot \gamma_k / q(\theta_k(t)).
\]

This comes from a derivation with respect to time of Eq. (3).

**Lemma 2.**

\[
\frac{\partial j_k(0,t)}{\partial t} = (r + \lambda + \delta + \sqrt{\epsilon_\kappa}) \left[ \left( \frac{\gamma_k}{q(\theta_k(t))} - \frac{\gamma_k}{q(\theta_k(t))^2} \right) \right]
\]

where \( \gamma_k / q(\theta_k(t)) = (p_k(t) - \theta_k + \lambda \int_0^{\tau(t)} G(\Omega) d\Omega) / (r + \lambda + \delta + \sqrt{\epsilon_\kappa}) \) and \( \theta_k(t) \) is the steady-state value of labor market tightness when the price and the reservation values are precisely at \( p_k(t) \) and \( R_k(t) \).

**Lemma 3.** Combining Lemmas 1 and 2, we obtain a dynamic equation for \( \theta_k(t) \):

\[
\frac{\partial \theta_k}{\partial t} = \frac{r + \lambda + \delta + \sqrt{\epsilon_\kappa}}{\eta(\theta_k(0)/q(\theta_k(t)))} \left[ \frac{\gamma_k}{q(\theta_k(t))} - \frac{\gamma_k}{q(\theta_k(t))^2} \right] \quad (A17)
\]

where we have used that \( \eta = - \theta_k q'(\theta_k)/q(\theta_k) \). Note also that \( 0 \leq \eta \leq 1 \) and that \( \eta \) depends on \( \theta_k \) except when the matching function is Cobb-Douglas in which case \( q(\cdot) \) is iso-elastic.

**Lemma 4.** Log-linearizing Eq. (A17), we obtain the forward-looking dynamic equation governing the law of motion of tightness:

\[
\frac{\partial \theta_k}{\partial t} = (r + \lambda + \delta + \sqrt{\epsilon_\kappa} [\theta_k(t) - \theta_k^*]). \quad (A18)
\]

**Lemma 5.** Log-linearizing Eq. (3) and using Eq. (A18), we obtain the forward-looking dynamic equation governing the law of motion of \( R_k(t) \):

\[
\frac{\partial \theta_k}{\partial t} = (r + \lambda + \delta + \sqrt{\epsilon_\kappa} [R_k(t) - R_k^*]).
\]

### A.6. Existence and uniqueness of a saddle-path

After a few manipulations around the steady-state, we can show that the system is described by a matrix of deviations around steady-states as:

\[
\begin{pmatrix}
\frac{\partial e^n}{\partial t} \\
\frac{\partial e^o}{\partial t} \\
\frac{\partial \theta_n}{\partial t} \\
\frac{\partial \theta_o}{\partial t} \\
\end{pmatrix} = \begin{pmatrix}
-J_{D^n} & \phi_n' & 0 & 0 \\
-\delta & -\phi_n' & 0 & 0 \\
0 & 0 & \Lambda & 0 \\
0 & 0 & 0 & \Lambda \\
\end{pmatrix} \begin{pmatrix}
e^n(t) - e^n* \\
e^o(t) - e^o* \\
\theta_n(t) - \theta_n* \\
\theta_o(t) - \theta_o* \\
\end{pmatrix}. \quad (A19)
\]

The signs of eigenvalues in this 8x8 dimensional system are easily derived in the text: we have 4 positive ones (the As) due to \( R \) and vacancies being forward-looking, and four negative ones due to the convergence of employment and unemployment that are predetermined. Hence, around this steady-state we are in the linear situation analyzed in Blanchard and Kahn (1980) in the case of linear difference equations, and generalized in Buttel (1984) to continuous-time linear differential equations: we have four predetermined variables, four “jump variables” and three exogenous variables \( p_k(t) \) and \( v(t) \). The information set is such that all agents form the right expectations and know the law of motion of exogenous variables. The number of “explosive solutions”, i.e. the number of eigenvalues with a positive real part, is exactly equal to the number of “jump variables”, while the number of eigenvalues with a negative real part is equal to the number of predetermined variables. There is thus, around the steady-state, a unique convergence path to the steady-state. See Sargent and Wallace (1973) for a very early analysis of the “stability” of economic system with two eigenvalues of different sign and the interpretation of the convergence to the unique saddle-path. This generalizes the analysis of continuous-time dynamics of the one sector exogenous labor supply matching model by Pissarides (2000, chapters 1–2) to the case of a two-sector matching economy with time-varying labor supply in each economy.

A.7. Remarks on the dynamics

Several important remarks on the dynamics follow:

1. There may be cases in which the old sector is so unprofitable that no firm creates any vacancy. Inspection of Eq. (3) shows that this may occur if \( R_0(t) \) is negative. What happens however in this case is that all existing firms would disappear, as the support of operating costs is positive. This means that, for a new firm a vacancy would be filled immediately, making infinite profits because \( p_c = \lambda \delta_0 - \lambda^d - \rho \) applied to \( Y_0 = 0 \) is infinite. Thus, zero tightness is never an equilibrium along our transition paths.

2. There is another corner solution: it may be that \( R_t \) is temporarily at the value of the upper bound of the support of \( \Omega \), in which case the rate of job destruction is at its exogenous component: \( J_{D_0}(t) = \delta + \sum_{t} \). We encounter such cases in several instances depending on the parameterization.

3. Most of our analysis of the uniqueness of the transition path is carried out in a neighborhood of the steady-state. It is well known that models with saddle-path such as the Ramsey growth model may well converge to a corner solution with zero capital stock, depending on initial conditions. Here, we have not explored formally the possibility that, far away from the steady-state, the system converges to other corner steady-state with zero vacancies in one of the two sectors. We have however not encountered such a possibility in the simulations. Furthermore, we believe that the argument that a zero vacancy rate also implies that all existing firms disappear is sufficient to rule out such a possibility.

4. At time \( t_0 \), the discrete negative jump in \( R_0 \) implies that there is a sudden, discrete inflow into unemployment and thus a discrete negative jump in employment in the old sector, by the quantity \( \sum_{t} \). In the modern sector, \( R_0 \) rises, meaning less job destruction and more hires. There is no discontinuity in \( e_e \) and \( u_e \) which are both state variables with no jump: the matching process smoothes the adjustment. After the shock, all stocks \( c_b \) and \( u_k \) are continuous, and thus can be considered as predetermined: the uniqueness result of the saddle-path thus holds for \( t=t_0 \). A discussion of such discrete jumps in otherwise predetermined variables can be found in Mortensen and Pissarides (1999).

A.8. Extensions: firing costs, retraining and early retirement

The empirical part of Section 3 revealed additional facts that the benchmark model presented in the main text does not address, but that extensions to it can rationalize. The objective of these extensions is to provide a quantification of the causes of the unemployment divergence of Poland and Estonia in the period of analysis. Hence, we will limit them to the simplest possible ones in order to be able to account for the facts. Following our discussion, we will extend the model to account for three labor market policies: employment protection, early retirement and retraining. Since we do not do any welfare analysis here, we will not endogenize the inflows into retirement or retraining: exogenous policy parameters are enough for the objective of the section that is mainly descriptive.

First, we assume that termination of an employment relationship has some cost \( F \) for the firm. This termination cost is seen as a pure tax to dismissals, and is not a transfer to the worker. See Mortensen and Pissarides (1999) for a discussion of this modelling choice. Whatever the wage rule (exogenous or as a share of the price of the intermediate good), the termination cost will thus have no direct consequence on wages. It however affects the job termination decision as firms will prefer to keep a temporarily unproductive relation in waiting for better times, as long as the present-discounted value of the job is above \( -F \). The job destruction rule (2) thus becomes:

\[
p_c(t) = -W_0 - R_0(t) + \frac{\lambda}{\tau + \delta + \lambda + \sum_t} \int_0^\tau G(\Omega) d\Gamma + (\tau + \delta + \lambda + \sum_t) F = 0,
\]

while the job creation rule remains unchanged.

Second, the existence of skills specific to sectors or occupations has a direct implication: workers endowed with such skills are a priori immobile, unless adequately retrained. What happens to a worker when a job destruction shock strikes? We have so far adopted an extreme assumption: the absence of mobility, which implies that this worker can only look for a job in the same sector. An alternative assumption is that some workers retrain. They can do so at some cost which we denote by \( \tau \). To simplify, we assume that retraining occurs only after job displacement, i.e. both newborn workers with vocational training and employees in the traditional sector do not retrain. Let \( T(t) \) be the total number of displaced workers in the old sector who retrain, with \( 0 \leq T(t) \leq \tau J_{D_0}(t) - \delta \delta_0 \). Thus, \( \tau \leq 1 \) is the fraction of displaced workers who retrain through active labor market policies. We will assume that labor market policies are rationed, so that both \( T(t) \) and \( \tau \) are policy determined: this is consistent with the observation that most countries have developed retraining programs targeted at specific groups of workers (long-term unemployed, younger workers, unskilled, displaced workers from a specific sector). Trainees are thus randomly selected from the pool of displaced workers.

In addition, workers may wish to endogenously retire if the value of being out-of-the-labor force is greater than the value of being in the labor market. In labor markets with frictions, it is always the case that in equilibrium workers with a job are better off than without a job, which implies that the crucial retirement decision margin concerns displaced workers in the old sector. In reality, pension systems are typically non-linear and discontinuous: prior to the legal age of retirement, workers are normally not eligible for pensions benefits. This would suggest that early retirement is not an endogenous option. When pre-retirement is observed, this is actually driven by government decisions: the government contributes to social security taxes for specific groups of the population, which allows them to leave the labor force. We thus naturally assume in the model that the total number of pre-retired workers is policy determined, and exogenous to worker’s decisions. The components of early retirement (level of benefits and total number of allowances) are fixed such that workers who are offered early retirement will accept it.

To keep symmetrical notations, the flow number of pre-retirees is denoted by \( \rho(t) = \pi (J_{D_0} - \delta) \delta_0 \), being \( \pi \leq 1 \) the fraction of pre-retirees.

Appendix A.3 reports Bellman equations describing workers’ utility in each state (employment \( W_0 \) and unemployment \( U_0 \), for \( k = n, o \)). If we denote by \( \tau \) the value of early retirement under a government-sponsored retirement scheme and by \( \tau \) the value of early retirement when it is not government-sponsored, the participation constraints in the pre-retirement program and in the retraining program imply that in equilibrium \( \tau U_0 > \tau \tau \) and \( U_0 - C \geq U_0 \). The first inequality insures that no worker quits the labor force without being sponsored by the government. Note also that, for workers to prefer the new sector to the old sector, as assumed ex-ante, we simply require that \( U_0 \geq U_0 \) which must necessarily be satisfied if \( U_0 - C \geq U_0 \) holds.

The full dynamics of state variables with positive \( P(t) \) and \( T(t) \) is described in Appendix A.2 in Eqs. (A7)-(A9).

---

26 A highly interesting empirical and theoretical question would be to understand why retraining is not more frequently the outcome of an individual decision. A combination of both credit constraints affecting displaced workers (who cannot afford paying training costs \( C \) and myopia of workers currently employed in declining sectors might explain why governments typically take care of retraining.
Appendix B. Additional Table and Graphs

![Unemployment rates](image)

**Fig. 6.** Poland-type economy after a reallocation shock. Initial stock of vocational education is 66%; early retirement flows is 5% of the flows of displaced workers; wages are rigid (\(w = 2/3, \beta = 0.1\)); employment protection \(F = 2.5\); no retraining. X-axis: quarters since initial shock.

**Table 6**


<table>
<thead>
<tr>
<th>Transition rates</th>
<th>E-E</th>
<th>E-U</th>
<th>E-I</th>
<th>U-E</th>
<th>U-I</th>
<th>I-E</th>
<th>I-U</th>
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<tbody>
<tr>
<td><strong>Estonia</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>All</td>
<td>3.34</td>
<td>1.46</td>
<td>1.38</td>
<td>10.76</td>
<td>6.63</td>
<td>2.97</td>
<td>1.59</td>
</tr>
<tr>
<td>Older Females</td>
<td>2.36</td>
<td>1.02</td>
<td>1.26</td>
<td>8.86</td>
<td>6.70</td>
<td>2.63</td>
<td>1.60</td>
</tr>
<tr>
<td>Older Males</td>
<td>2.92</td>
<td>1.50</td>
<td>0.79</td>
<td>8.55</td>
<td>4.85</td>
<td>2.58</td>
<td>1.23</td>
</tr>
<tr>
<td><strong>Poland</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>All</td>
<td>1.48</td>
<td>1.64</td>
<td>1.57</td>
<td>8.64</td>
<td>6.62</td>
<td>1.43</td>
<td>1.85</td>
</tr>
<tr>
<td>Older Females</td>
<td>0.37</td>
<td>0.45</td>
<td>3.10</td>
<td>3.04</td>
<td>12.74</td>
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<td>0.42</td>
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<tr>
<td>Older Males</td>
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<td>4.84</td>
<td>9.33</td>
<td>0.81</td>
<td>0.46</td>
</tr>
</tbody>
</table>

Note: Yearly transition rates. Older females are between 50 and 60 years of age. Older males are between 50 and 65. Weighted statistics. E: Employment; I: Inactivity; U: Unemployment; E-E refers to job movers.

References


